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The high-energy critical minimum in elastic electron scattering by argon

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Abstract. The position of high-energy critical minimum in elastic electron-argon scattering was investigated both experimentally and theoretically. Differential cross-sections (DCSs) were measured as a function of both incident electron energy (40−150 eV) and scattering angle (40−126°), in small steps around the critical minimum. The position of the high-energy critical minimum in elastic electron-argon scattering was experimentally found to be at 129.4 ± 0.5 eV and 119.4° ± 0.5°. To cover the energy and angular ranges of the present experiment, relevant relativistic ab initio calculations were carried out, based on the Dirac-Hartree-Fock method with the exchange calculated exactly. Target polarization is described by an ab initio potential taken from relativistic polarized orbital calculations. The calculated position of the high-energy critical minimum is 118.0 ± 0.5 eV, 118.9° ± 0.3°. It was shown that even slight difference of fixed scattering angle close to the critical point could affect significantly the energy dependent DCS. Discussion of behavior of DCS in the vicinity of the critical minimum was performed including convolution analysis in both energy and angle.

PACS. 31.15.Ar Ab initio calculations – 34.80.Bm Elastic scattering of electrons by atoms and molecules

1 Introduction

In recent years a number of new theoretical results for elastic electron-argon scattering has been reported [1–4]. The calculations were performed to obtain the angular position of differential cross-section (DCS) minimum versus incident electron energy and/or the positions of critical minima, which are defined by the points where DCS attains its smallest value as a function of both incident electron energy (E_0) and scattering angle (θ). This minimum, being a singular point on DCS surface with precisely defined two parameters (E_0, θ), gives possibilities for very sensitive comparison of experimental and theoretical results. The direct scattering amplitude, which is normally dominant to define DCS, becomes very small in the region around the critical minimum and, hence, theoretically obtained critical minima are very sensitive to the chosen method and to the number of different processes included in calculation. Buhring [5] was the first to bring attention to critical points and there are quite a few results for elastic electron-argon scattering reported in the second half of previous century [6–10]. Panajotović et al. [11] reported the first detailed experimental investigation of two lower-energy critical minima in elastic electron-argon scattering, which could be compared efficiently to the theoretical results. They have measured only angle dependent DCSs at fixed incident energies up to 100 eV. Experimentally obtained positions of the high-energy critical minimum (above 100 eV) have been reported only preliminary by Lucas and Liedtke [7] and Kessler et al. [8]. Moreover, there are no published experimental results for DCS minimum positions versus incident energy between 100 eV and 150 eV nor there are detailed DCS measurements with sufficient resolution in this energy region, relevant for investigation of critical minima. On the other hand, there is a considerable disagreement of the published positions of this minimum, of the order of about 10 eV and 5° (excluding the results of Lucas [9], which scatter much more). Finally, an accurate determination of critical points in elastic electron-atom scattering would be also beneficial for either angular or energy calibration in experimental measurements (e.g. Kollath and Lucas [12]).

Present experimental investigation of the high-energy critical minimum in elastic scattering of electrons by argon was performed by independent measurements of DCS as a function of either incident electron energy or scattering angle. This provided a consistent DCS data set in the (E_0, θ) region around the critical minimum and absolute calibration of this data set according to the single point.

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Hence, the obtained DCS surface as well as the critical minimum position was not influenced by previous measurements. Although the angular and energy resolution were not sufficient to obtained the real depth of the critical minimum, the obtained experimental results should be of use to localize the high-energy critical point in argon and also to investigate a very strong coupling between energy and angle dependence of DCS around it. Present experimental results were compared with the theoretical DCSs obtained by relativistic ab initio calculations. This theoretical method gave a very good agreement with the previous experimental results of Panajotović et al. [11] at lower incident electron energies [1,3]. However, the present calculated critical energy appeared to be in somewhat larger disagreement with present experiment.

2 Experiment

In short, experiment consists of a crossed beam spectrometer with an electron gun, electron energy analyzer and channel electron multiplier as a detector. The effusive atomic beam is formed using a stainless steel, nonmagnetic needle placed perpendicularly to the incident electron beam. The electron gun with a hairpin thermoelectron source is used to produce a well collimated electron beam. This gun can produce up to 1 mA of incident beam current in the energy range from 20 eV to 500 eV. The analyzing system consists of simple four-element electron lens followed by double cylindrical mirror analyzer (DCMA). The double μ-metal shield reduces the Earth and other magnetic fields to less than 2×10^{-7} T. Background pressure was approximately 3×10^{-7} mbar. During the measurements, all the parameters were kept in the range where the contribution of double electron scattering was negligible. Also, the influence of the effective path length correction was found to be negligible in the covered angular range from 40° to 130° according to Brinkmann and Trajmar [13].

In the case of energy dependent DCS measurements, a consideration was taken of the most important energy dependent factors. For a broad energy domain (90 eV to 150 eV) the potential of the last (focusing) electrode of the gun was tuned as the incident energy was varied. This was monitored by a Faraday cup. For the analyzer section, the transmission function of the four-electrode electron lens was maintained constant over a wide energy region by an appropriate choice of lens potentials. Additionally, the optimal electron gun lens potentials and analyzer lens transmission function were confirmed by electron trace simulation programs (Milosavljević et al. [14,15]). The detection efficiency was kept constant by keeping the difference between the analyzer cylinder potentials constant, i.e. keeping the analyzer transmission energy constant during the measurement.

The energy scale was checked with respect to the minimum position of the energy dependent DCS for elastic electron-argon scattering at the fixed scattering angle of 70°. This minimum is well determined by DCS measurements of Cvejanović and Crowe [16] that were performed as a continuous function of energy. Also, the same has been roughly revealed by absolute normalized angle dependent measurements of Panajotović et al. [11], Srivastava et al. [17] and Williams and Willis [18]. Energy dependence of DCS at 70° is shown in Figure 1a. A good agreement between our DCS and previous results is observed at this scattering angle. The agreement of the minimum position of present DCS and the one obtained by Cvejanović and Crowe [16] is practically perfect, without any energy shift applied. Cvejanović and Crowe reported

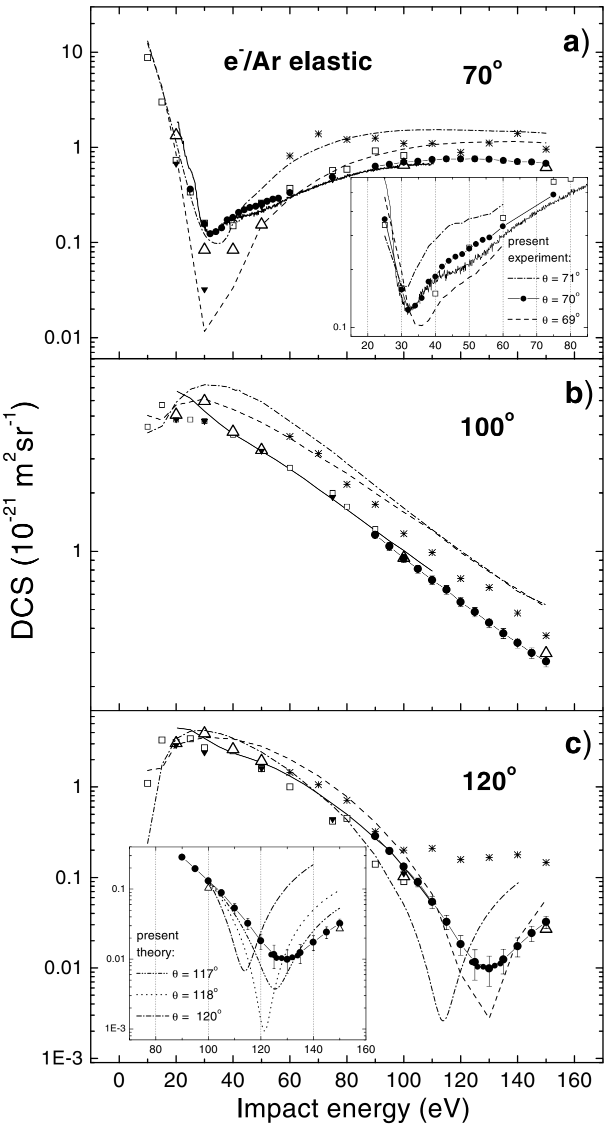

Fig. 1. Energy dependence of absolute DCSs for elastic electron-argon scattering at different fixed scattering angles. Experiment: (-•-) present, (□) Panajotović et al. [11]; (−) Cvejanović and Crowe [16], (▼) Srivastava et al. [17], (∗) Vušković and Kurepa [23], (△) Williams and Willis [18]. Theory: (− · − · −) present (− − −) Fon et al. [27]. Present experimental DCS are presented at 70.0° ± 0.6°, 100.0° ± 0.6° and 119.5° ± 0.5°. In the insert in (c), the theoretical DCSs are normalized to the experimental curve at 100 eV, in order to present more clearly the change of minimum position.

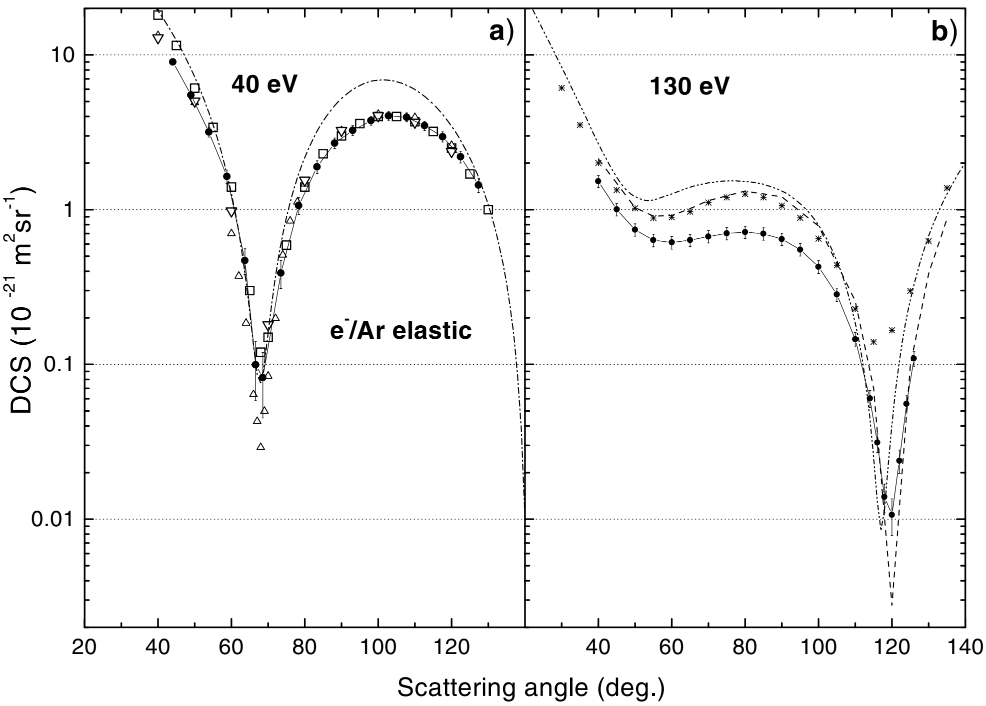

Fig. 2. Angular dependence of absolute DCSs for elastic electron-argon scattering at different fixed incident energies. The legend is the same as for Figure 1 except that (▽) represents experiment of Cvejanović and Crowe [16]. The upper limit of uncertainty of the incident energy scale for present experimental DCSs is ±2.5 eV.

the upper limit of uncertainty in the energy scale to be ±0.3 eV. Further uncertainty in our energy scale is due to the ±1° uncertainty in the angular position reported by these authors. According to present measurements (see insert in Fig. 1a), the position of minimum in energy dependent DCS shifts for 5 eV as the scattering angle is varied from 69° to 71°. Corresponding to energy and angular uncertainties in the work of Cvejanović and Crowe [16], an upper limit of uncertainty in our energy scale of ±2.5 eV is obtained. This is less than 2% of the obtained critical energy position.

The overall energy resolution of the system was found to be approximately 1 eV, which is more than sufficient to separate elastically scattered electrons from inelastically ones. The calculations predict that in the vicinity of the high-energy critical point, DCS is more sensitive to variation of scattering angle than to variation of incident energy. Also, the calculations of Walker [6] predict that with increasing the energy of the critical point, the angular spread decreases and the energy spread increases. Hence, we did not expect a very strong influence of low energy resolution on the energy dependent DCSs and so to the accuracy of the obtained position of the critical point.

The true zero of the angular scale was obtained for each measurements from the symmetry of inelastic scattering intensity between −20° and +20°. The angular resolution was estimated by comparing present angular distributions with the previous measurements at the incident energies where DCSs contain extremely deep minima. Such a minimum exists in elastic electron-argon scattering at 40 eV and 68°. The obtained angular depth was at least equal to those reported by Panajotović et al. [11] and Cvejanović and Crowe [16] (see Fig. 2a). The authors stated angular resolutions of ±1.5° and ±1.36°, respectively. Williams and Willis [18] performed a detailed investigation of this angular minimum and found that its depth did not change for acceptance angles less than 1.5° (resolution of ±0.75°). However, there are few published results in the energy region above 100 eV relevant for comparison. Additionally, we have investigated the angular resolution of the experimental system using the electron trace simulation programs. Finally, we found that our angular resolution is better than ±2.5°.

Present relative differential cross section at 100 eV and 100° is normalized with respect to the experimental DCS of Srivastava et al. [17]. This absolute DCS is also in good agreement with the value published by Williams and Willis [18]. All angle dependent DCSs at other energies are normalized at 100° by using experimentally obtained absolute normalized energy dependent DCS at 100° (see Fig. 1b). The relative energy dependent DCSs at 70.0° and 119.5° (shown in Figs. 1a and 1c) are then normalized at the incident energy of 100 eV.

3 Theoretical method

To obtain the wave function for the scattered electron with a given symmetry κ and energy E we solve the radial Dirac-Fock equation (Grant [19]) which can be written in atomic units as

$$\left(\frac{\mathrm{d}}{\mathrm{d}r}+\frac{\kappa}{r}\right)P_\kappa(r) = \{2/\alpha+\alpha[E-V_{fc}(r)-V_p(r)]\}Q_\kappa(r)+X_Q(r),$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}r}-\frac{\kappa}{r}\right)Q_\kappa(r) = -\alpha[E-V_{fc}(r)-V_p(r)]P_\kappa(r)-X_P(r), \quad (1)$$

where P_κ and Q_κ are radial parts of the *large* and *small* components of the Dirac wave-function and $\kappa = \pm(j+1/2)$ for $l = j \pm 1/2$ comprises the total angular momentum j and parity $(-1)^l$, α is the fine structure constant. In addition, V_{fc} is the relativistic frozen-core potential, V_p is the polarization potential; the two terms X_P and X_Q describe the exchange potential between the incident electron and bound electrons at the target.

Both, the exchange terms and the frozen-core potential V_{fc} are calculated from first principles by using the one-electron orbitals as obtained by the multiconfiguration Dirac-Fock (MCDF) program of Desclaux [20] with some modifications (Sienkiewicz and Baylis [21]). These terms are defined as

$$V_{fc} = -\frac{Z}{r} + \frac{1}{r}\sum_{j,k} a^k(s,j)Y^k(j,j;r)$$

$$crX_{P(\text{or } Q)} = \sum_{j,k} b^k(s,j)Y^k(s,j;r)P_j(\text{or } Q_j)$$

where index "s" refers to the scattered electron, Z is the nuclear charge and the sums are over electrons of the target atom. The radial function Y^k and the angular coefficients a^k and b^k are given by Grant [19].

The polarization potential V_p can be derived in perturbation theory as a second-order correction to the frozen-core approximation. In our present approach, it includes the dipole static term and is taken in a numerical form from the ab initio calculations of Szmytkowski [22] which were done with the relativistic version of the polarized orbital method.

From the solutions of the Dirac-Fock equations above, we obtain the phase shifts $\delta_l^\pm$ by comparison with the analytical form at large r,

$$P_\kappa(r)/r = j_l(kr)\cos\delta_l^\pm - n_l(kr)\sin\delta_l^\pm \qquad (2)$$

where k is the momentum of the incident electron, $j_l(kr)$ and $n_l(kr)$ are the spherical Bessel and Neumann functions, respectively. Here, δ_l^+ is the phase shift calculated for $\kappa = -l-1$ in equation (1) and δ_l^- that for $\kappa = l$. In the case of a relativistic scattering problem we have two scattering amplitudes: the direct one

$$f(\theta) = \frac{1}{2ik}\sum_l \{(l+1)[\exp(2i\delta_l^+) - 1] + l[\exp(2i\delta_l^-) - 1]\}P_l(\cos\theta) \qquad (3)$$

and the spin-flip one

$$g(\theta) = \frac{1}{2ik}\sum_l [\exp(2i\delta_l^-) - \exp(2i\delta_l^+)]P_l^1(\cos\theta). \qquad (4)$$

In equations (3, 4) θ is the scattering angle, while $P_l(\cos\theta)$ and $P_l^1(\cos\theta)$ are the Legendre polynomials and the Legendre associated functions, respectively. With these two scattering amplitudes, differential cross-section for elastic scattering is defined by

$$\sigma_{diff}(\theta) = |f(\theta)|^2 + |g(\theta)|^2 \qquad (5)$$

while the spin polarization cross-section is given by

$$S(\theta) = \frac{i[f(\theta)g^*(\theta) - f^*(\theta)g(\theta)]}{\sigma_{diff}(\theta)}. \qquad (6)$$

4 Results and discussion

The absolute energy dependent DCSs at the fixed scattering angles of 70°, 100° and 120° (present measurements at 119.5°) are presented in Figure 1. It should be noted that regarding previous results, only the DCSs of Cvejanović and Crowe [16] have been obtained as a continuous function of incident electron energy. Hence, our measurements are the first experimental study of the energy dependence of DCS for elastic electron-argon scattering extended up to 150 eV. Present experimental energy dependent DCSs at 70.0°, 100.0° and 119.5° were used for the check of energy scale, absolute normalization of data and precise determination of the critical energy, respectively. At 100° all presented DCSs descend smoothly over the incident energy range from 40 eV to 150 eV (the similar shapes have been obtained at the angles of 80°, 90° and 110°). However, the energy dependence of DCSs at 70° and 120° are more complex, with deep DCS minima that correspond to the positions of the critical points. Present measurements reveal the high-energy minimum at 120° that was theoretically predicted, but has not been experimentally confirmed. The only available gas-cell measurements of Vušković and Kurepa [23] do not have sufficient resolution to reveal the minimum. Present DCS agree very well with two data points at 100 eV and 150 eV reported by Williams and Willis [18].

The low-energy minimum position of the present calculated DCS at 70° agrees very well with the experiment (Fig. 1a), while at the scattering angle of 120°, the calculated high-energy minimum is shifted towards lower values (Fig. 1c). However, in the insert in Figure 1c, it could be seen that the agreement of the calculated high-energy minimum position becomes much better at a few degrees smaller scattering angles. Hence, in the vicinity of the critical point, the slight variation in scattering angle could cause a significant shift in the position of minimum in the energy dependent DCS. Having this in mind, it would be interesting to compare angle dependent DCSs in this region. In Figure 2, the angular dependence of absolute DCSs at the 40 eV and 130 eV is presented. Both of these energies are close to the critical point. While at 40 eV agreement in low-angle minimum position is achieved by all experiments and theory, at 130 eV the high-angle minimum of the present calculated DCS is indeed slightly shifted towards smaller scattering angles. Also, there is a disagreement in absolute DCSs, calculated value being for a factor 2 larger than experimental one at local maximum around 80°.

Projection of DCS surface on the (E_0, θ) plane is presented in Figure 3. The experimental DCS reaches the absolute minimum at about 130 eV and 120°. In order to determine the exact position of the critical point, we have performed measurements in small ranges of impact

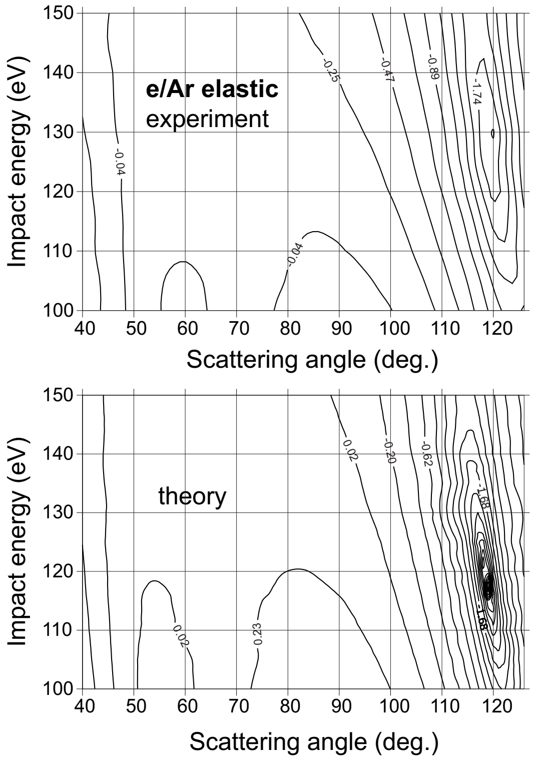

Fig. 3. Projection of 3D surface of DCS(E_0, θ) for elastic electron-argon scattering on the (E_0, θ) plane in the vicinity of the high-energy critical point. Numbers correspond to the logarithm of the DCS values in units of 10^{-21} $\mathrm{m^2\,sr^{-1}}$.

energies and angles around it. The procedure was to measure DCS in the range of scattering angles from 117° to 123° with small increments of 1°. For each scattering angle fixed, the incident electron energy was varied from 124 eV to 132 eV in increments of 2 eV. According to these measurements, the position of the critical angle was determined to be 119.4° ± 0.5°. The critical energy value of 129.4 ± 0.5 eV was then obtained by fitting the energy dependent DCS at the fixed angle of 119.5° (see Fig. 1c). The theoretical contours in Figure 3 show the critical minimum to be at about 118 eV and 119°, although they seem to indicate a double minimum structure in the vicinity of the critical point. This becomes more noticeable with coarse mesh of the calculated results, while additional minima almost vanish for very fine data resolution around the critical point. There is no evidence of double or triple high-energy minimum in the present experimental results nor it was previously reported.

In Figure 4, the angular position of high-angle DCS minimum versus incident electron energy is presented. Previous experimental [11,17,18] and theoretical [24–26] data are used to illustrate behavior in a large energy range from 5 eV to 150 eV. The present experimental points were independently extracted from the angle dependent measurements. The procedure was to fit the relative angle dependent DCS around local minimum to Legendre polynomials by least square method to obtain precisely the minimum position. The observed dependence is practically linear above 100 eV, except for the small plateau in the vicinity of the critical point. Hence, the position of high-angle DCS minimum appeared to have very week energy dependence in the vicinity of the critical point. On the other hand, the energy position of the DCS minimum is very sensitive to the scattering angle, as it was shown before.

The only previous experimental values for the high-energy critical point in elastic electron-argon scattering are preliminary results of Lucas and Liedtke [7]: 130±3 eV; 120.1° ± 0.5°, and Kessler et al. [8]: 132.3 ± 0.3 eV; 120.90° ± 0.05°. These are in good agreement with the present experimental result, although the latter critical values (reported with smaller errors) [8] are slightly higher. In the same contribution, Kessler et al. presented theoretical DCS for elastic electron-argon scattering. These data were calculated using programs supplied by Walker [6] and represent solutions of the Dirac equation for a relativistic Hartree-Fock potential, including exchange. Obtained critical position: 126.1 eV; 118.9°, is in better agreement with present experiment than the one by Kessler et al. [8]. Also, the most recent theoretical result of Kelemen [2]: 126.33 eV; 118.12° agrees very well with present experiment, as well as the calculated critical energy of Khare and Raj [10]: 125.8 eV; 117.33°, while the critical point of Paikeday [4]: 120.5 eV; 117.3° has somewhat lower values of both energy and angle. These critical positions are compared to the present results in insert of Figure 4. Lucas [9] performed a detailed investigation of critical points in elastic electron-argon scattering by using either theoretically or experimentally obtained phase shifts of different authors as the input. However, the calculated critical points scatter substantially and are not presented in the figure, although some of them are very close to the present ones. Finally, the only published results above 100 eV, relevant to show the behavior of DCS in the vicinity of the critical point, are those of Fon et al. [27]. The DCS surface obtained by R-matrix method appeared to agree very well with the present experiment, although the data have been published with a rather large energy steps, so the exact position of the critical point cannot be extracted with sufficient accuracy.

Present calculated critical point being: 118 eV; 118.9°, differs from experimental one, regarding critical energy value. The obtained difference is equivalent to the disagreement between energy dependent DCS minima at the angle of 120° (Fig. 1c), which was considered above. Although the present theoretical method showed better agreement with experiment for two lower-energy critical minima [3], it was indicated that in the case of the high-angle critical minimum the agreement with the experiment of Panajotović et al. [11] was also not as good as in the case of the low-angle minimum. According to both experimental and theoretical contour plots in Figure 3, the gradient of DCS is much greater along the angular axis than along the energy axis in the vicinity of critical point. Therefore, only slight shift in angular scale in high-angle region of calculated results could cause a significant disagreement of DCS data around the critical point.

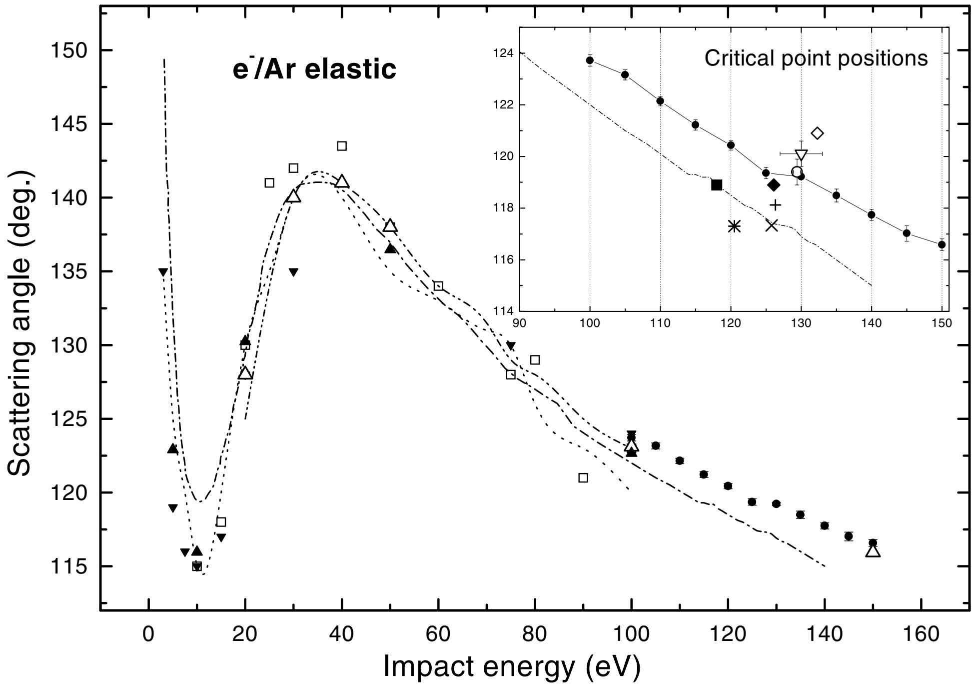

Fig. 4. Position of high-angle DCS minimum versus incident electron energy for elastic electron-argon scattering. Experiment: same as for Figure 1. Theory: (– · – · –) present, (– · · – · · –) Bartschat et al. [24], (···) Nahar and Wadehra [25], (▲) Haberland et al. [26]. Available positions of high-energy critical point are presented in the insert. Experiment: (◯) present, (◊) Kessler et al. [8], (▽) Lucas and Liedtke [7]. Theory: (■) present, (+) Kelemen [2], (∗) Paikeday [4], (×) Khare and Raj [10], (♦) Kessler et al. [8].

The experimental investigation of DCS around the critical minimum was limited by both angular and energy resolution of the apparatus. Therefore, a profound analysis of the obtained results should include the convolution of theoretical DCSs with experimental resolution. Since we investigate the behavior of DCS surface around critical point, only the comparison of experimental and convoluted theoretical results in this region is of interest, regardless the discrepancy of the obtained critical values. The convolution of theoretical results with experimental resolution in both energy and angle is presented in Figure 5. In Figure 5a, the calculated DCS minima at several fixed incident energies around critical value are convoluted with angular resolution of 2.5° and compared to the appropriate experimental DCSs. Although the theoretical DCS is rather deep around the critical minimum, the convoluted surface appeared to agree very well with the experimental one. The global picture presented in Figure 5a is made more readable in the insert by shifting the angle dependent DCS curves. Moreover, the full convolution of angle dependent DCS at 118 eV (calculated critical energy) around minimum, with the resolution of 2.5° is presented in the insert in Figure 5a as well. The convoluted curve agree perfectly with the experimental DCS at 130 eV (practically critical energy), considering error bars. On the other hand, the convolution in energy, even with the resolution of 2 eV, does not lift up appreciably the DCS minima points (see Fig. 5b). Hence, the energy resolution of about 1 eV of the present experiment should not influence the investigation of the high-energy critical minimum, which was already stated earlier. Moreover, the large signals obtained by using only electron gun provided much more accurate measurements of DCSs in the region around deep minima.

According to the convolution analysis presented in Figure 5, the main experimental limitation on reaching the real depth of the high-energy critical minimum in elastic electron-argon scattering is imposed by finite angular resolution of the apparatus. Since the DCS is extremely sensitive to small changes in angle around the critical point (see Fig. 3), the slight shift of critical angle value, which is due to this angular uncertainty, could cause the shift of a few electron volts of measured critical energy. A very precise experimental investigation of the high-energy critical minimum, comparable with calculations, demands extremely high angular resolution. On the other hand, there is always a trade between high resolution and signal intensity in electron spectrometers. In measuring of critical points where signals are extremely small, it is difficult to obtain simultaneously both conditions.

Errors and averaging

The absolute angle dependent DCS values have been calculated as a weighted mean of at least three independent measurements. Absolute errors are defined by statistical error, according to the Poisson's distribution and by normalization error that is the consequence of an angular uncertainty of about 0.6° of the normalization point (100 eV, 100°). The latter error depends on the incident energy since the shape of angular dependent DCS (normalized with respect to the energy dependent DCS at 100°) changes with different E_0. In the case of energy dependent DCS measurements, we have limited the energy interval so that the possible errors due to the variations in both incident electron beam and transmission function are within statistical errors. Finally, as we have normalized our results to the absolute DCS value of Srivastava

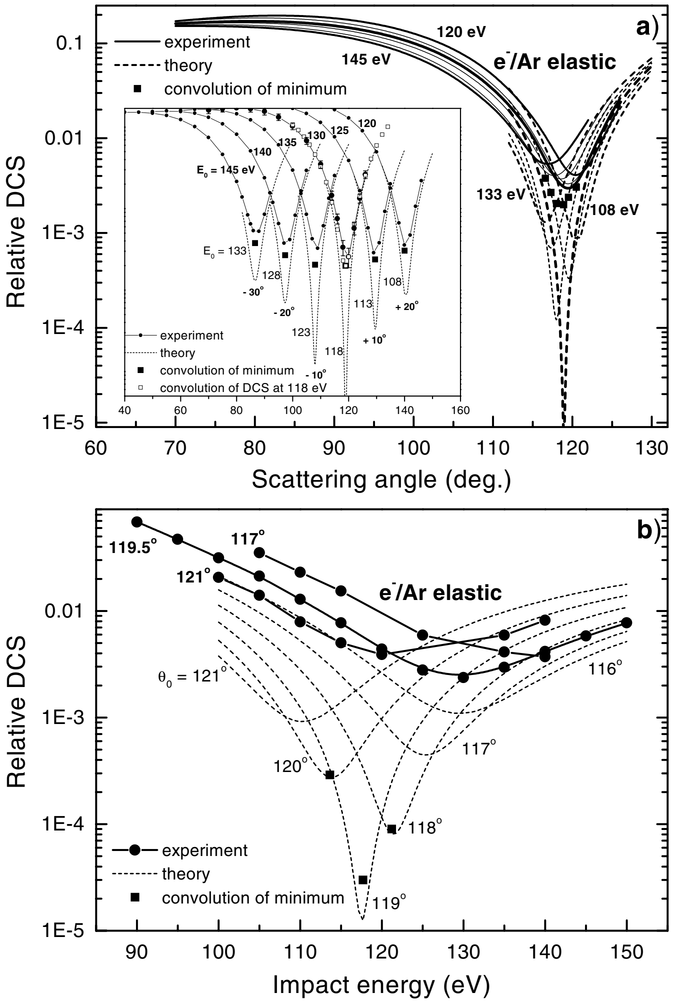

Fig. 5. Comparison of convoluted theoretical results in both angle and energy with the experimental ones. In order to obtain more effective comparison, the absolute values of two data sets are scaled by normalizing angle dependent DCSs at critical energies to the same value at 80°. (a) Convolution in angle with the resolution of 2.5°. In the insert, the DCS curves are shifted in angle to provide more readable comparison. (b) Convolution in energy with the resolution of 2 eV. The legend is given in the figure.

et al. [17], the 20% errors quoted by these authors should be applied as well.

The error of the zero angular position has been calculated for each angle dependent measurement and it was always less than 0.4°. Hence, in the case of energy dependent measurements at the fixed scattering angle, it was assumed that the uncertainty of the zero angular position should remain less than 0.4° as well. On the other side, for determination of the DCS high-angle minima positions, a fitting procedure of experimentally obtained points was applied. In this case, the fitting errors have been accounted as well and we have estimated them to be less than 0.3°. The final minima positions were also calculated as a weighted mean of several measurements.

The very critical point is defined by two parameters, namely critical incident energy and critical scattering angle. These two values depend on each other and, hence, the energy and angular uncertainties of the obtained critical point are coupled. For the critical point obtained by our measurements, we have estimated the errors to be 0.5 eV for the critical energy, and 0.5° for the critical angle. These errors are dominantly defined by the low energy resolution and the uncertainties of the scattering angles as well. Finally, the possible shift of the critical energy due to the uncertainty of the incident energy scale calibration should be less than 2% of its value (see Sect. 2).

5 Conclusion

A detailed investigation of critical point in elastic electron scattering by argon atoms in the energy range between 90 eV and 150 eV and angular range between 40° and 130° was performed. Both angle and incident energy dependent relative DCSs were measured separately in small energy and angular steps around the critical values. The absolute normalization of experimental data was performed at the single point (100 eV, 100°). For the first time, the high-energy minimum at 120° was experimentally determined by measurements performed in small energy steps. The position of the third critical point in elastic electron-argon scattering was experimentally found to be at 129.4±0.5 eV and 119.4° ± 0.5°. This agrees very well with most of the available published calculations. The only previous experimental values of this critical point have been preliminary reported by Lucas and Liedtke [7] and Kessler et al. [8]. The agreement is very well, although the letter result (which is more precise) has slightly higher critical values. In addition, the position of high-angle DCS minimum as a function of the incident electron energy is experimentally obtained in small steps above 100 eV. It appeared that in the vicinity of the critical point, the minimum position does not change appreciably.

To cover the energy and angular ranges of the present experiment, relevant relativistic ab initio calculations were carried out. The calculated position of the high-energy critical minimum is 118.0 ± 0.5 eV, 118.9° ± 0.3°. The theoretical approach is based on the Dirac-Hartree-Fock method. Exchange between incident and target electrons is calculated exactly. Target polarization is described by an ab initio potential taken from relativistic polarized orbital calculations.

Generally, the calculated DCSs agree well with the experiment. However, they show some shift of the energy dependent DCS minimum at the angles and energies close to the third critical point. Nevertheless, it was shown that even slight shift of the fixed scattering angle affects significantly the energy-dependent distribution in the angular and energy region close to the critical point. Finally, the theoretical results were convoluted with the experimental resolution in both energy and angle, in order to access the accuracy of the results and to investigate more deeply the behavior of DCS around it. The convolution in angle of theoretical points around the critical minimum gave a very good agreement with the appropriate experimental results. On the other hand, the energy resolution of the present experiment appeared to had negligible influence on the obtained critical point position.

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